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# Dynamic Pricing to Maximize Mobile Network Revenue (Almadar Aljadeed: Case Study)

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## Abstract

Cellular networks are characterized by a scarcity of resources, particularly bandwidth and frequency spectrum. This leads to frequent and significant congestion. Some solutions were proposed to alleviate the congestion problem with installing new infrastructure. However, these solutions often imply either an increased system complexity or a significant degradation of the quality of service. This paper studies one solution to this problem to modify the way in which the user can access to the network to fit the available resource. This leads to the principle of dynamic pricing. The user behavior affects prices depending on the network usage, where high price during periods of congestion will make some users postpone their calls or shorten them. Also, the user can try to move to another cell to obtain a cheaper price, because the price varies in terms of location of the user in the city or countryside. The proposed solution is able to make a better use of the available bandwidth, and provide a greater revenue to the network operator as well as improving the quality of service to the users.

***Keywords*** – Dynamic price, Demand and Supply, Cellular system, Linear and non-linear pricing, Call blocking.

## I. INTRODUCTION

Pricing plays an important role for the design, operation, and management of communication networks. Traditionally, engineers have designed communication services without worrying about how these services should be priced. This may be due to the fact that in early time, there are very few monopolistic service providers in the market provisioning very few services. These service providers are guaranteed enough profit, even with simple flat-fee pricing schemes. For example, flat-rate pricing, i.e., a constant price for the network services, is very effective in stimulating new applications development, but it is no longer suitable for an environment with an increasing demand for network resources.

The recent advanced technologies bring major changes to the communication industry. The wireless communication market has experienced an incredible growth as well the demands for both wire line and wireless data services. Both of these service have been growing exponentially due to the increasing popularity of internet multimedia applications, mobile devices, and social networks.

The enormous growth of the cellular phone market has implied a great demand for radio resources. Since the availability of the radio bandwidth is limited, new solutions to increase the available capacity of radio systems need to be found. Different approaches can be used, from cell splitting and frequency reuse to overlapping cell layers result in smaller cell clusters. Hence, urban areas now have far more cells per square kilometer than rural ones, and different dynamic channel allocation techniques are adopted.

However, the difference between peak and off-peak demand for mobile services tends to be very significant, with only a few very busy hours during the day and much quieter periods at other time. Thus, meeting the peak demand for mobile

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services is costly and the resulting network capacity remains under-utilized most of the time, which these methods often imply either an increased system complexity or a significant degradation of the quality of service.

The effect of price on users' demand for fixed line calls was investigated by previous researches. They observed a positive correlation between the increase in price and the reduction in the number of calls made by customers. This was true even for flat rate customers, when the reduction in call frequency had no effect on their bill. They also observed that the estimates that customers made about their telephone usage followed a lognormal distribution [1,2], i.e. while a few customers underestimated their usage, a large majority overestimated it and their estimates were widely scattered.

A solution was found based on dynamic pricing techniques, i.e., on pricing strategies where the cost of network operator charges per time unit depends on the network usage and is dynamically adapted to the network status, where it can make a better use of the available bandwidth, and provide the desired quality of service to the users as well as a greater revenue to the network operator. It is intuitive that the trend of users demand during the day can be modified by imposing high rates in the correspondence of peak-traffic time periods and low rates when large radio resources are available. Thus, making prices dependent on the network usage can be an efficient solution to network congestion problems.

The dynamic pricing scheme given in [2] is claimed to be very effectively to regulate demand whilst offering a sensible means for revenue optimization, where this presents predicted GSM network behavior with a dynamic pricing regime. It suppresses their demand for the service when the network is busy and increase their usage at off-peak times. This led to reduction of blocked calls by up to 30%. The results of this simulation indicate a strong correlation between the shape of the price function, the number of blocked calls in the system because users will have a choice as to whether to proceed with a call or not.

In [3], the user demand function has been modeled and the call duration of the service price. By using standard Markovian techniques, an optimal linear pricing policy is derived in order to obtain a transparent and easily controllable pricing strategy. When compared with a flat-rate policy, where a constant price for the network services is fixed, the proposed solution is able to provide a better quality of service to the users.

The study in [4] tries to resolve a problem of congestion in the case of both GSM and GPRS networks to give better results if it can be combined dynamic pricing and more traditional approaches; where a detailed traffic model for both GSM and GPRS networks is given and implemented in an event-driven simulator. It was proved that a combination of the best Call Admission Control (CAC) schemes and dynamic pricing gave even better results.

In [5], dynamic pricing policies allow network service providers to charge per unit time depending on the availability of network resources; The goal is to regulate the behavior of users and significantly improve network management. Using standard Markovian techniques to represent the evolution of the system, the approach creates an ideal linear pricing chart, which can be easily calculated and controlled. This solution is able to provide better quality of service to users as well as increase revenue for the network operator.

Dynamic pricing has been also studied in [6] for the efficient allocation of spectrum in wireless networks with selfish users. A belief-based dynamic pricing approach has been proposed to improve overall spectrum efficiency while maintaining the incentives involved for users based on double auction rules. Simulation results showed that the scheme could approach optimum performance using limited overhead.

The research in [7] presents a dynamic pricing policy and new rate control model-based congestion control scheme in price, where it is introduced a dynamic pricing function as one of the important metrics to local rate control of neighboring nodes. The result confirms that dynamic pricing based congestion control is effective for alleviating congestion, and also can improve network performance, such as throughput, fairness and the ratio of packet loss.

The resource analysis approach in [8] studies the value of dynamic pricing by comparing it with several other reasonable approaches, including static pricing and choice-based availability control. This study shows that dynamic pricing can lead to a significant across the board revenue lift in the order of 1%–6%, and concludes that dynamic pricing approaches should be implemented whenever possible in practice.

In [9], an optimal dynamic pricing scheme is designed for maximizing the expected long-term revenue. A sequential dynamic pricing scheme of a monopoly mobile network operator in the social data market is proposed. The proposed scheme exploits the network effects in the mobile users' behaviors. The authors propose a modified sequential pricing policy to ensure fairness among mobile users in terms of their individual utilities. The study analytically demonstrates that

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the proposed dynamic pricing scheme can help the operator gain greater revenue and users achieve higher total utilities than those of the baseline static pricing scheme.

The rest of this paper is organized as follows. Section II presents dynamic pricing overview. Section III gives different possible pricing models supported by mathematical expressions. An efficient dynamic model is proposed in Section IV where a closed form formula for pricing algorithm is derived. Different call holding time models are given in Section V, while a strict user utility function is proposed in Section VI. Simulation, discussions and conclusions are drawn in Section VII and Section VIII respectively.

## II. Overview of Dynamic Pricing

The dynamic pricing model is the concept of selling the same product at different prices to different groups of people, also referred to as surge pricing, demand pricing, or time-based pricing. It is a pricing strategy in which businesses set flexible prices for products or services based on current market demands. Businesses are able to change prices based on algorithms that take into account various external factors, including current market demand, season, supply changes, and price limits. Simply, dynamic pricing is a strategy in which product prices continuously adjust, sometimes in a matter of minutes, in response to real-time supply and demand.

Dynamic pricing is a common practice in several industries such as hospitality, tourism, entertainment, retail, electricity, and public transport. Each industry takes a slightly different approach to dynamic pricing based on its individual needs and the demand for the product [10]. There are multiple types of dynamic pricing methods that a business can use. One of these methods, dynamic peak pricing, in which prices are higher during peak seasons for the product, and dynamic segmented pricing, where this type of dynamic pricing depends on the geographical area. As the market changes, supply and demand can increase or decrease the price of a product.

### *A. Traditional Tariffs*

Prices for the services may change depending on the time of day, day of the week, or time elapsed for the service. Therefore, some service providers may have a set of service prices to fully describe the service fee. However, it is still often considered static, because even if prices change, prices are not based on current conditions [11].

A single service may have multiple definitions, separated by some restrictions (such as time or day). These constraints regulate the engines to which tariffs are applied. In this way service providers can encourage users to use during specific time periods. This is required in wireless networks to encourage users to use the network during peak periods.

One of the types of restrictions is tariff restrictions, such as maximum fees. It regulates the maximum fees that can be paid for a consecutive period. This can be a maximum fee per minute, hour, day, range, or services (no time limit). Moreover, the units of date and time can have different meanings depending on the wording. For example, a day can refer to 24 hours, or a calendar day (also determined by the time of the daily reset). Continuously, a week can refer to 168 hours, 7 calendar days, or a calendar week (weekend regression). Finally, there are more ways to determine the month, especially since each month does not have the same number of days.

Tariffs can also be divided into two types: regular and static [12]. Regular tariffs are the rates that are charged depending on the elapsed or expected service time. On the contrary, static tariffs are the prices which are charged in full for a predetermined period, and the tariff is usually fixed. A tariff can be defined as an hourly, daily, weekly, monthly or another predefined period. In addition to the two types of tariff, service fees may be charged in advance, which is known as prepaid services. It is intended for use in wireless networks with the same idea.

### *B. Dynamic Pricing*

Dynamic pricing means that service prices change based on current conditions, in contrary to traditional service which changes according to old conditions. Moreover, this change can occur monthly, weekly, daily, or even on an hourly basis. However, the important factor is that the changes can be made in real-time based on current conditions, compared to the traditional tariffs, which often are based on old conditions [13].

Dynamic pricing can be used to ensure that service occupancy is kept at an optimal level [11]. Dynamic pricing is considered to be a part of smart service. Moreover, dynamic pricing does not simply mean a change of the pricing strategy, but rather that prices will be based on demand and supply. The differences between truly dynamic pricing and a varying

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pricing strategy are how often and long it takes to change service rates. Because changing service rates today takes too many steps. Service rates are relatively static or at least change very slowly. The reason for the slow rate of changes undergoes number of steps. The steps start with declarations the new rats, meetings and required involve meetings and conversations to understand the service, get feedback from the community about these new rates, make final decisions based on this feedback, and then finally announcing the new rates to the public. In contrast, dynamic pricing would automate many of the steps, while skipping some of these steps. Therefore, being able to keep up with fluctuating demands.

## III. PRICE MODEL SCHEMES

### *A. Static Pricing Model*

Traditionally, wireless service providers have supported static pricing schemes, where a service provider buys the user a bundle of usage minutes per month and a fixed set of services that can be accessed throughout the duration of the contract. The difference between peak and off-peak demand for cellular services tend to be very significant with only few busy hours during the day and much quieter periods at the other times, on the other hand, network capacity remains underutilized during off peak period. The number of cells in large cities has almost reached its maximum and reducing the size of the cells further would add more overheads than the benefits [14].

Therefore, it is important to have a mechanism that will improve overall utilization and performance of the network based on the traffic load. In wireless networks, pricing has been suggested as an effective mean to resolve the allocation of the scarce resources to the users. Researchers have used pricing integration with call admission control where dynamic peak hour price is applied only when traffic load increases beyond the optimal value. Queueing models are used where users are categorized as premium users and normal users [15].

Static or flat pricing analysis assumes that the network will sell all of call arrival rate at a fixed price. Linear pricing deals with a network that can engage in price discrimination (or price differentiation), that is, charging different rates depend on the call arrival rate. The goal of price discrimination is to increase network revenue by reducing congestion.

A price is a charge that is associated with one unit of service usage. As an example, assume a mobile service providers operate a two-part tariff of the form $a + bx$, where $a$ is a monthly fixed charge (some time called "access charge"), $x$ is the number of minutes of calling per month and $b$ is the price of the call per minute. $x$ is also known as the amount of consume. So, the linear price function can be expressed as a function of incoming arrival rate.

$$P_L \propto \lambda_{in} \quad \rightarrow \quad P_L = P_{min} + K\ \lambda_{in}\ , \tag{1}$$

where $P_L$ is linear price of a call (unit price), $K$ is proportionality constant, $\lambda_{in}$ is incoming call arrival rate, the average number of calls that are incoming. It gives how many calls have to be handled or put on hold during a particular period (call/sec), and $P_{min}$ is the minimum price applied at no load condition (unit price).

This simple model is illustrated in Figure 1. It is clear from Equation (1) and Figure 1 that linear pricing cannot be optimized, since the price increases as the traffic increases. Linear pricing plan is the best choice for callers whose calling rates are less than ($\lambda_o$), while flat rate is better if their calling rate is more than that value.

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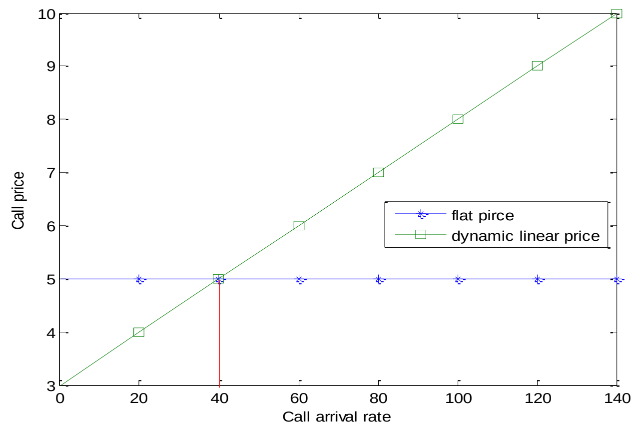


**Figure 1.** *Flat and linear pricing strategy.*

Advantages and disadvantages of flat rate pricing can be summarized as below. Flat pricing has the great advantages of simplicity in implementation and avoids source of contention about the bill between customer and supplier. The bill is always predictable. The main disadvantage of flat rate pricing is the absence of any direct relation between price and cost. There is no distinctive to prevent users generating an excessive traffic. Flat pricing is unfair in the case of vastly different usages.

Finally, flat pricing is associated with best effort service, there is no way to allow any user who wishes to pay more to avoid congestion or blocking. For the case of linear pricing, the strategy is also simple to manage by the service provider. It keeps a marginal profit on each usage period regardless of how many are the frequency of usage. The drawback of linear pricing model is that it doesn't provide adequate incentive for customers to stay long time using the service, they always try to terminate the service as short as possible. This off-course reduces the total revenue of the service provider and cannot estimate the total profit.

### *B. Dynamic Pricing Model*

Dynamic pricing involves the use of economic and behavioral strategies to control congestion as well as increase resource utilization in mobile wireless network. The price for utilizing mobile wireless network resources is determined dynamically according to the network load. The price is increased when the network load is high to reduce the demand for mobile wireless network resources and decreased when the network load is low so as to increase the demand of mobile wireless network resources. As such, dynamic pricing is used to promote a rational and efficient use of mobile wireless network resources by influencing the users' behavior [16].

Current wireless networks use static pricing schemes, users are charged with constant or fixed rate throughout the time of the day. The major advantage of this scheme is that the billing and accounting processes are simple. However, the price is independent of the current state of the network or any dependence is fixed and is based on decisions that have been made statically and may not correspond to the actual system conditions. Hence, such systems cannot avoid congestion, cannot react effectively to the dynamic changes in the traffic conditions, and contributes unpredictable variation to the network usage and conditions.

This study has introduced dynamic linear and nonlinear closed loop pricing model for peak and off peak periods. The developed mechanism allocates resources based on dynamic pricing where the price of a call varies based on traffic load on a network. One of the most important advantages of dynamic pricing will be to reduce call blocking probability and increase QoS. In dynamic pricing, high demand during peak period will spread over to off peak period which will result in improving overall utilization of the system.

Let us consider a simple quadrature function as a non-linear model in the form

$$P_{NL} = -\alpha\, \lambda_{\text{in}}^2 + \beta\, \lambda_{in} + P_{min} \qquad (2)$$

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where the negative sign is inserted to make sure that the curve is concaved downward and so it should have maximum value. The parameter $\alpha$ and $\beta$ are design factors and should be chosen carefully to maximize the profit and satisfy the customer quality of service. ($\alpha, \beta > 0$), $P_{NL}$ is non-linear price. Differentiation Equation (2) with respect to $\lambda_{in}$ and equating to zero, we get

$$\frac{dP_{NL}}{d\,\lambda_{in}} = -2\alpha\,\lambda_{in} + \beta = 0 \;\rightarrow\; \lambda_{in} = \frac{\beta}{2\alpha} \tag{3}$$

Substituting (3) into (2), the maximum revenue can be written as

$$P_{NL,max} = \frac{\beta^2}{4\alpha} + \mathrm{P_{min}} \tag{4}$$

Equation (4) shows that the revenue is increased over the flat pricing by the factor $\frac{\beta^2}{4\alpha}$. This factor can be controlled by the network operator according to traffic network congestion and to user QoS requirements. The non-linear model shown above is valid only from $P_{min}$ up to $P_{NL,max}$ values, i.e.,

$$\mathrm{P_{min}} \le P_{NL} \le P_{NL,max}$$

$P_{NL,max}$ is achievable at the upper bond of network capacity which can be found by substituting Equation (4) into Equation (2) to have;

$$P_{NL,max} = -\alpha{\lambda_{in}}^2 + \beta\lambda_{in} + P_{min}$$

$$P_{min} + \frac{\beta^2}{4\alpha} = -\alpha{\lambda_{in}}^2 + \beta\lambda_{in} + P_{min} \quad\Rightarrow\quad -\alpha{\lambda_{in}}^2 + \beta\lambda_{in} - \frac{\beta^2}{4\alpha} = 0 \quad , \quad \alpha,\beta > 0$$

Solving for $\lambda_{in}$ yields to

$$\lambda_{in} = \lambda_{cap} = \frac{-\beta \pm \sqrt{\beta^2 - \frac{\beta^2}{\alpha}\alpha}}{-2\alpha} = \frac{\beta}{2\alpha}, \tag{5}$$

where $\lambda_{cap}$ can be considered as the bottleneck capacity of the network. Now we can find a relationship between $\alpha$ and $\beta$ as

$$\alpha = \frac{\beta}{2 \times bottleneck\ capacity} = \frac{\beta}{\lambda_{cap}}, \tag{6}$$

Figure 2 illustrates this non-linear model. Non-linear price model is plotted versus traffic load for different values of α.

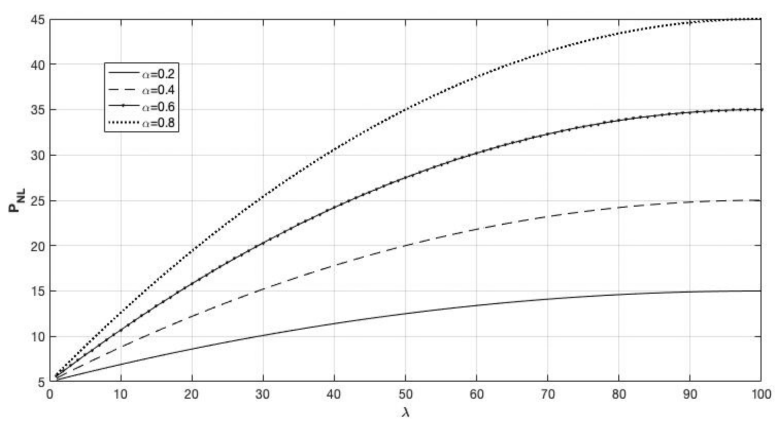


**Figure 2**. *Second order model for pricing.*

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The network service provider may assign different classes of QoS for different users as their desires. Higher QoS users (i.e. higher α) will be charged more than lower QoS users.

The above model is for a single user, it can be extended to multiple users and write the maximization problem as follows:

$$\begin{aligned} &\underset{\alpha_i,\lambda_i}{\text{maximize}} -\alpha_i\lambda_{in,i}^2 + 2\alpha_i\lambda_{cap}\lambda_{in,i} + P_{min} \\ &s.t \qquad \alpha_i > 0 \\ &\qquad\quad \lambda_{in,i} < \lambda_{cap} \end{aligned} \tag{7}$$

## IV. PROPOSED MODEL OF DYNAMIC PRICING

Most of the current mobile networks charge their customers with flat rate or based on the time of the day/month. These schemes are characterized by simplicity in billing and in accounting processes. Such scheme does not consider the current state of the network and cannot avoid traffic congestion. It is also unable to react with dynamic state of the network or volatile variation of the network usage. Therefore, this paper proposes a model which will be able to integrate the congestion and call admission control together to overcome this issue. A block representation of this model is shown in Figure 3.

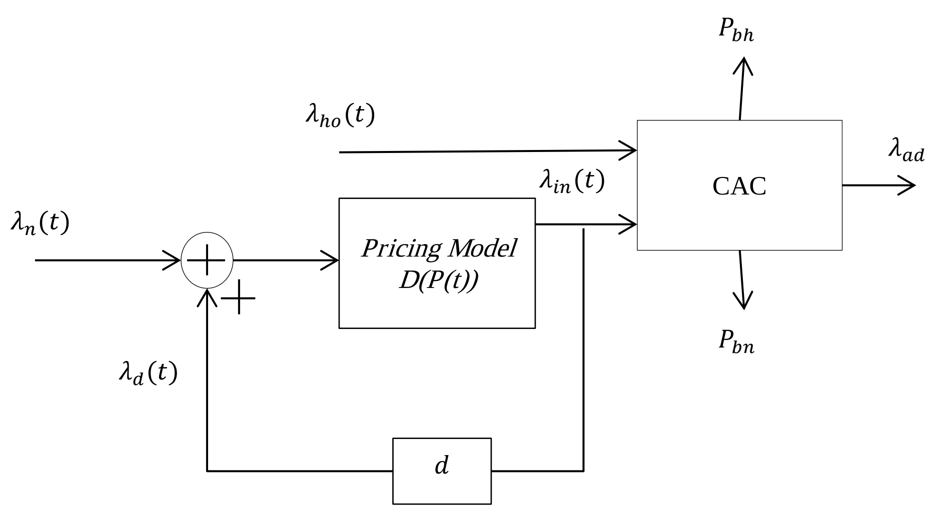


**Figure 3.** *Block diagram of the proposed model.*

The block diagram parts are described below:

1- $\lambda_{ho}(t)$ represents the handover call arrival rate, this is the call rate generated from other neighboring congested cells. In wireless networks the handover call rate has always a priority over any new arrival call. This means that the handover call has more important than new calls. This is because establishing a new call requires an unequipped channel which sometimes cannot be provided and hence the call might be dropped or blocked, while the incoming handover call is already in progress.

2- $\lambda_n(t)$ represents the new call arrival rate. The call inter-arrival is a function of the time of the day. Statistically, this can be generated and modeled using Poisson distribution. The probability of $k$ arrivals in interval of length $\lambda$ is given by the Poisson distribution:

$$P(\lambda; k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad k = 0,1,2,\ldots \tag{8}$$

where: $P(\lambda; k)$ is the probability of $k$ arrivals in interval of length $\lambda$. λ is the call arrival rate (call/second).

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Blocking in wireless network happens when a network is fully occupied and unable to accept further calls. Due to blocking in wireless network, calls are either queued (but not lost) or are lost (all calls made over congested network). The probability of the block is given [16]:

$$P_b(t) = \begin{cases} 0 & when\ \lambda_{in} < \lambda_{cap} \\ \dfrac{\lambda_{in}(t) - \lambda_{ad}(t)}{\lambda_{in}(t)} & when\ \lambda_{in} > \lambda_{cap} \end{cases} \quad (9)$$

where: $\lambda_{cap}$ is the call arrival rate corresponding to full capacity of the system (call/second). Full capacity is the total number of clients a cell can receive. $\lambda_{ad}(t)$ is admitted call arrival rate at time $t$ (call/second).

The expected number of users ($M$) that can be admitted to the system is a function of $\lambda_n(t)$, i.e., $M = f(\lambda_n(t))$.

3- $\lambda_d(t)$ is the delayed traffic. This represents the traffic generated by some users those are not willing to accept extra charge during peak hour price. They might prefer to wait or delay their calls at this moment of network condition until the price is reduced and the network becomes less congestion. From Figure 3, we can note that the output of pricing model block is

$$\lambda_{in}(t) = D(P(t)).(\lambda_n(t) + \lambda_d(t)) \quad (10)$$

4- $\lambda_{in}(t)$ is the traffic rate entered to the Call Admission Control (CAC) block. There is a certain value, say $\lambda^*$ which represents the optimal traffic that the system can provide with the given quality of service and guarantees that the cell will not be in congestion condition.

5- D(P(t)) is the pricing scheme block that should be adapted according to the instantaneous traffic condition $\lambda_n(t)$.

6- $P_{bn}$ and $P_{bh}$ are the blocking probability of new call and the blocking probability of handover call respectively. The total probability of blockage can be modeled as a weighted sum of these probabilities; i.e.,

$$P_b = aP_{bn} + bP_{bh} \quad (11)$$

$P_b$ reflects the acceptable metric of the user satisfaction or say quality of service. It is characterized as the user utility function. $a$ and $b$ are design parameters that depend on accepting or rejection new calls and handover calls, where: $0 \le a \le 1,\ 0 \le b \le 1$. As we have shown in point (1) above, $b$ should be chosen greater than $a$ to give more importance for handover calls. System overload conditions cannot be avoided because users' requests are independent and maybe selfishly. They do not consider the current network traffic and available resources. If we let each user requests the resources those maximize his/her level of satisfaction (QoS), then the total utility of the subscribers will decrease. Therefore, some adjustment must be done to provide incentive for users to react in a way that improves the total utilization and to satisfy their quality of services. This adjustment could be achieved by dynamic pricing.

The demand function describes the behavior of users to the change of price. Many demand functions have been used in literature [17]-[19]. In this study, we will adopt the demand function proposed in [19]:

$$D(P(t)) = e^{-(\frac{P(t)}{P_{min}}-1)^2} \qquad P(t) \ge P_{min} \quad (12)$$

where $P(t)$ is the price charged to users at time $t$. Equation (12) denotes the percentage of users those will accept the new price. Using Equation (10) and Equation (12) we can derive the price that should be applied at time $t$ in order to satisfy the desired quality of services. So, we can write:

$$e^{-(\frac{P(t)}{P_{min}}-1)^2} = \frac{\lambda_{in}(t)}{\lambda_n(t) + \lambda_d(t)} \rightarrow \left(\frac{P(t)}{P_{min}} - 1\right)^2 = \ln\left(\frac{\lambda_n(t) + \lambda_d(t)}{\lambda_{in}(t)}\right)$$

$$P(t) = P_{min} + P_{min}\sqrt{\ln\left(\frac{\lambda_n(t) + \lambda_d(t)}{\lambda_{in}(t)}\right)} \quad (13)$$

For optimal price, we substitute $\lambda_{in}(t)$ by $\lambda_n^*$ . So, Equation (13) becomes:

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$$P(t) = P_{min}\left(1 + \sqrt{\ln\left(\frac{\lambda_n(t) + \lambda_d(t)}{\lambda_n^*}\right)}\right) \tag{14}$$

Or as in closed form:

$$P(t) = \begin{cases} P_{min}, & \lambda_n(t) \le \lambda_n^* \\ P_{min}\left(1 + \sqrt{\ln\left(\frac{\lambda_n(t) + \lambda_d(t)}{\lambda_n^*}\right)}\right), & \lambda_n(t) > \lambda_n^* \end{cases} \tag{15}$$

It can be concluded from Figure 3 that the carried traffic $\lambda_{ad}(t)$ can be expressed as:

$$\lambda_{ad}(t) = \min(\lambda_{in}(t), \lambda^*) \tag{16}$$

This is to guarantee the user satisfaction.

The term of the square root in Equation (15) represents the extra price that will be applied to users those willing to use this plan of payment to continue their calls and maintain a satisfied QoS.

## V. CALL HOLDING TIME MODEL

The average call duration is an important measurement in telecommunications traffic that reflects an average length of telephone calls transmitted on telecommunication networks. In most of previous studies, the holding time of call is assumed to be independent of the price. The behavior of users in general is that they will tend to shorten their call as the price is raised, shortness will depend on the price being charged.

For this reason, it is important to investigate the impact of the call duration on the price. This also will reflect on the total revenue of the network operator. Three models of the variation of holding time with price will be presented and compared.

### *A. Fixed Call Holding Time Model*

This model supposes that the holding time of a call is constant and it does not depend on price. This model is used in [18], and it is given as:

$$H\big(P(t)\big) = H_0 \tag{17}$$

where $H_0$ is a constant call holding time (sec).

### *B. Exponential Decreasing Holding Time Model*

In this model, the holding time of the call will be expressed as exponential decreasing function with price. This can be written as:

$$H\big(P(t)\big) = H_0 e^{\gamma\left(1-\frac{P(t)}{P_{min}}\right)} \tag{18}$$

where $\gamma$ is a design parameter and it should be positive value. This holding time model is commonly used in most of literatures.

### *C. Squared-Price Exponential Model*

In this model, the holding time of a call is assumed to be exponentially depended on the square of the price. Mathematically, this can be written as:

$$H\big(P(t)\big) = H_0 e^{\gamma(1-(\frac{P(t)}{P_{min}})^2)} \tag{19}$$

Figure 4 illustrates these three holding time models.

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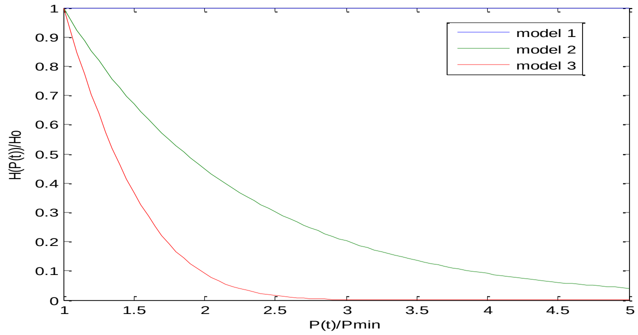


**Figure 4.** *Illustrates the three models for γ=0.8*

## VI. USER UTILITY FUNCTION

Utility function describes the user level of satisfaction. In terms of mobile services; it can be expressed by the quality of provided services. The higher the utility, the more satisfied by the user. So, it can characterize the sensitivity of the user to the change of QoS. Therefore, utility function can be used to model the willing of a user to pay money to satisfy his desires in terms of QoS.

Many utility functions are proposed by previous researchers, in this study, the utility function proposed in [19] will be modified as follows:

$$U_s(P_b) = \begin{cases} 1 - e^{15(P_b - 0.2)}, & 0 < P_b \leq 0.02 \\ 0 \quad , & P_b > 0.02 \end{cases} \quad (20)$$

The utility in this model degrades as $P_b$ increases. If the blocking probability drops to certain value, the user will not accept the service because his QoS is worse than his/her preferred level.

The revenue of the network operator is now calculated based on the previous models. This amount of revenue will vary with time (dynamic) and fluctuate according to the network condition. Mathematically, revenue can be modeled as:

$$R(t) = \lambda_{ad}(t).P(t).H(t).(1 - P_b) \quad (21)$$

## VII. SIMULATIONS AND DISCUSSIONS

Real-time traffic for model test is obtained from Almadar Aljadeed Company; the biggest mobile operator in Libya. This traffic is gotten from one important base station location for whole year. This data collection is then divided into four quarters, averaged and then normalized. This is to reduce the huge instantaneous obtained data.

Fixed, linear and non-linear pricing schemes are used in simulation, tested, analyzed and evaluated. Starting with a fixed rate, then setting a lower rate and applying it as the minimum call rate. This low price helps to attract users and motivate them to establish extra calls or/and make their call duration longer. This offers increases network utilization during off-peak times.

In dynamic price simulation, the call price per time is imposed as a function of arrival call rates. i.e. charging different rates depending on the call arrival rate. The rate is updated periodically based on the traffic load encountered and the probability of accepting call barring, and is then sent to the user so that the user can decide if they desire to use the network at that time at that price. The maximum call rate which is set appears at peak times (congestion period). This reduces the number of users by postponing or shortening their calls. The new price is sent to the user using the Broadcast Control Channel (BCC), so that the range is from minimum to maximum.

Each cell has the ability to support a certain number of calls (λ), when this number increases, congestion occurs in the network, making the network unable to receive new calls. The prevailing rate in the network is applied to the call once it is

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accepted by the network. When the call ends, its price is calculated and used as a function of the call's arrival price and so on for all calls.

The parameters that are used throughout this analysis are summarized as follows. The cellular cell is assigned a total of 40 channels in which two of them are preserved as guard channels. Each call requires only one of these channels to provide service. The average call holding time is assumed to have exponential distribution (model in Equation (18) is used) with mean 180 sec. The normal charging rate for users using channels ($P_{min}$) is chosen 3 dirhams per minute. The charging rate for users accepting dynamic charge depends upon demand function descended in Section IV and it is broadcasted upon subscribers to be announced. The parameters $a$ and $b$ in Equation (11) are chosen as $a = \frac{1}{3}$ and $b = \frac{2}{3}$. This is to give more probability to accept the handover call when a decision should be taken between a new call or handover call. This concept is explained in Section IV. $P_{bn}$ and $P_{bh}$ are chosen to have typical values of 1%.

The data traffic is recorded for one complete year (2020). The data is normalized to its peak value. The traffic distribution structure can be used to analyze the pattern of the call generation and manage it during peak and off-peak periods in a day. It can be also used to understand how the implementation can be done to optimize the network resources and maintain efficiency by scattering proportion of calls using pricing mechanism.

Figure 5 shows the call arrival rate in which the year is divided into four months so that each month is an average of three months.

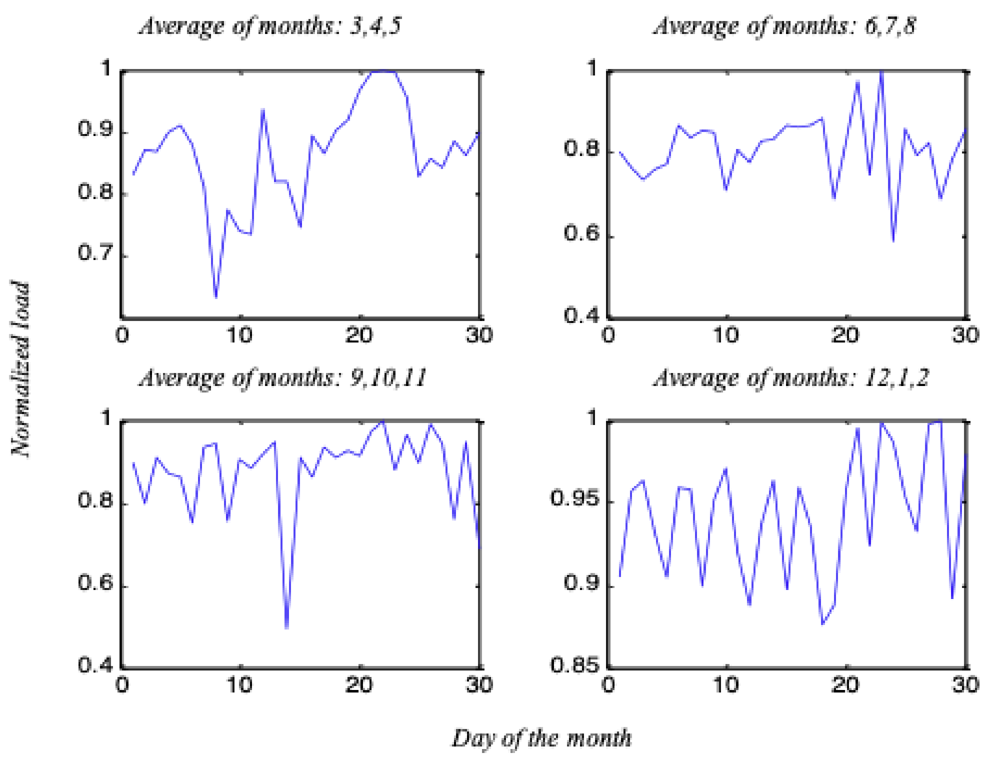


**Figure 5.** *Normalize traffic load over months.*

Figure 6 shows that the call price is higher for non-linear pricing scheme than that for linear pricing scheme. This can reduce congestion by controlling the flow of the calls in peak periods. This intern will optimize the usage of the spectrum.

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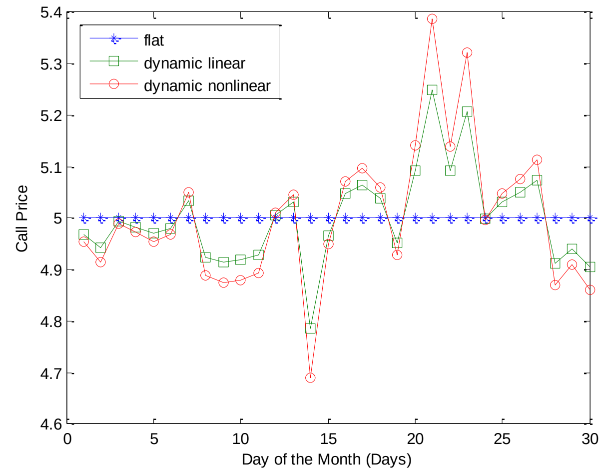


**Figure 6.** *Variation of call price with day of the month.*

This also can be noticed overall the year as depicted in Figures 7. In all months non-linear pricing performs better revenue utilization.

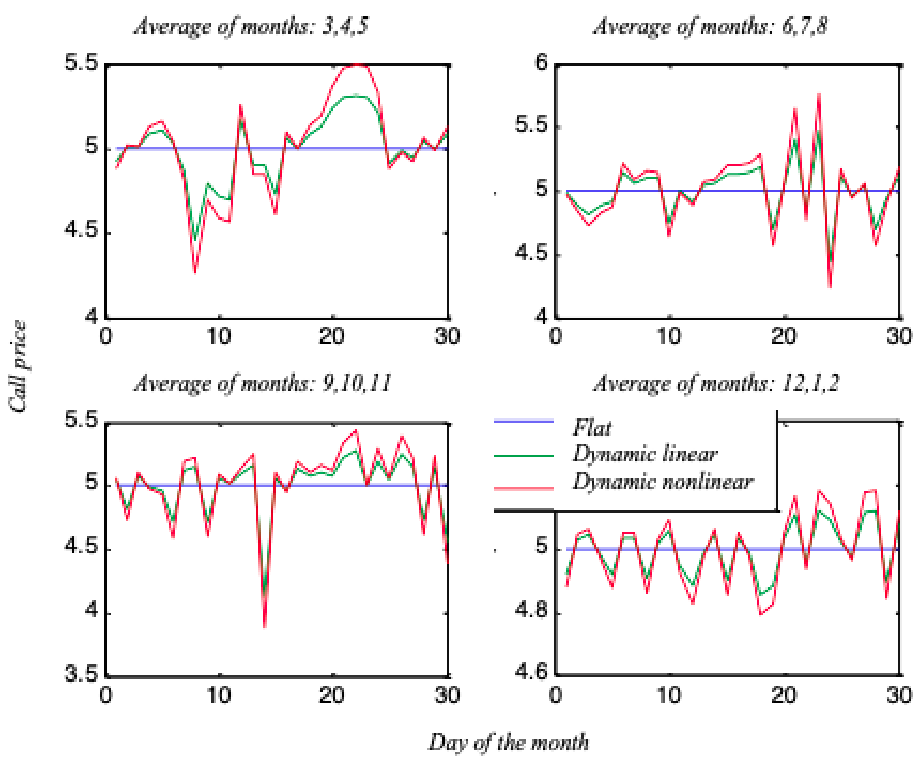


**Figure 7.** *Variation of call price with day of the month.*

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Figures 8 highlights the impact of static pricing that currently exists and dynamic pricing on revenue per day. The static pricing system has the worst revenue per day as compared to the other pricing schemes. By using non-linear pricing scheme, the return is better than linear scheme at peak days.

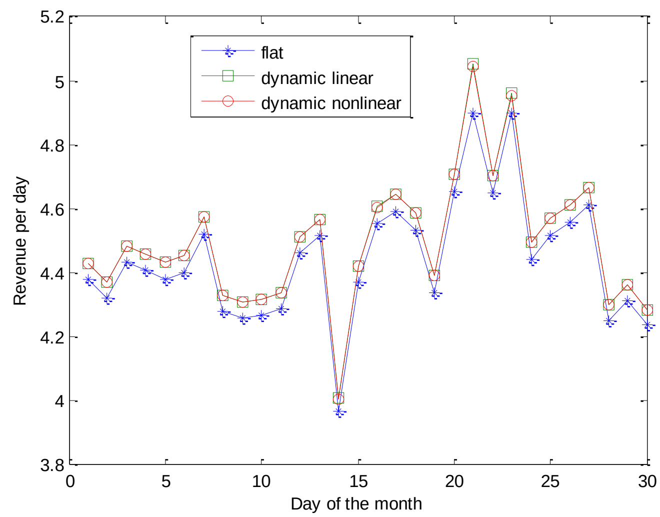


**Figure 8.** *Revenue per day in static and dynamic pricing.*

Table 1 below summarizes a comparison of static scheme, dynamic linear and nonlinear schemes in terms of values obtained for requested calls, served calls, blocked calls, blocking ratio, fulfillment ratio and revenue. Average system fulfilment is determined as a ratio of total calls served and maximum calls that the system can support.

Table 1 *Comparison of static scheme, dynamic linear and nonlinear scheme.*

| Scheme | Requested calls | Admitted calls | Blocked calls | Block % | Fulfillment % | Revenue (LD) |
|---|---|---|---|---|---|---|
| Flat | 7124 | 6840 | 284 | 3.98 | 62.2 | 2462 |
| Dynamic linear | 7748 | 7613 | 135 | 1.74 | 69.2 | 2653 |
| Dynamic nonlinear | 7812 | 7754 | 58 | 0.74 | 70.5 | 2691 |

## VIII. Conclusion

In this study, dynamic pricing based on an analytic approach is studied and analyzed, where the price per unit time changes in real time according to the network load, in order to make better use of the available bandwidth. Making variable fees dependent on network usage is an effective solution to network congestion problems, it affects the user behavior and allows to control network operating conditions. The study compares between different pricing schemes, starting with using static pricing and then determining models for dynamic pricing. Real traffic load is obtained from one of national mobile network operators. This traffic load is then applied to the proposed model. Extensive calculation and evaluations are performed. Price is raised when network load is high, this insures demand reduction. The pricing plane is announced to the customers to get their permissions to resume the call or terminate it. For the case when the demand is low, the price is kept to it is

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nominal value. This approach will reduce the demand on network resources. This system spreads the high demand during the peak period to the off-peak period which leads to the improvement of the overall usage of the network. The analysis in this paper presents a dynamic linear and non-linear closed-loop pricing models. The obtained results indicate the effectiveness of the system in reducing congestion in cellular networks while achieving quality of service and effective use of network resources. Conversely, the high price at peak times made some users delay or shorten calls. With this solution, the distribution of calls becomes better and the network receives more benefits. The network became less congested, and therefore better quality. Using dynamic non-linear schemes achieve better performance than the results obtained using a static and dynamic linear model. Finally, it is noticed that the actual traffic does not follow the exact Poisson distribution. The traffic varies randomly throughout the day, months and the year and it has one or more maximum values at peak hours.

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